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RATIO, DIFFERENCE, REGRESSION, AND JACKKNIFE ESTIMATION METHODS

IN AUDITING: A SYNTHESIS ANALYSIS*

by

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A B S T R A C T

Nine alternative estimation methods used in sampling survey have been reviewed to indicate the conditions of using alternative estimation methods in audit sampling. A set of accounts receivable data with 250 observations was used to do some simulation studies. It is found that the regression and difference methods were the most acceptable methods to be used in audit sampling.

I. Introduction

Stettler (1966), Hall (1967), McCray (1972), Ernst and Ernst (1975) and others have shown that client reported item values and reported balance associated with particular accounts can be used to improve an auditor's estimate of such accounts. However, ratio and difference estimation methods commonly used by accountants to estimate the accounting figures may not be the best suited for this purpose. It is well-known that ratio and difference estimation methods are only two out of several estimation methods developed by statistician [see Cochran (1963) and Yates (1960)]. Therefore, the appropriateness of the above-mentioned two estimation methods for auditing should be subjected to careful re-examination. After investigating the behavior of several major statistical estimations in sampling accounting populations, Neter and Loebbecke (1975, 139-140) have indicated the importance of examining additional estimators. The main purposes of this paper are to show that ratio and difference methods are only two special cases of a more general estimation method and to investigate the necessity of replacing the unbiased ratio estimation method for the biased ratio estimation methods in audit sampling. In this paper, data associated with accounts receivable are used to re-examine the possible shortcomings of both ratio and the difference method on the basis of both the variance and mean square error [MSE] criteria.

The plan of this paper is to first introduce the subject. In the second section, seven different estimation methods are reviewed and the conditions for using each method are discussed. In the third section the potential bias associated with using traditional estimation methods in auditing is addressed. The possibility of using the Jackknife method to remove the bias and to reduce the MSE of sample estimates in auditing

is then explored. In the fourth section, one set of accounts receivable data with 250 observations is used to simulate the estimate total, the estimated variances and the mean square errors in accordance with 9 estimation methods defined in this paper. Two hundred runs of simulation results are used to demonstrate the relative advantages of 9 different estimation methods explored in this study. The sample procedure used to select the sample in the simulation is also discussed. Furthermore, implications associated with these empirical results are discussed. Finally results of this paper are summarized, and concluding remarks are indicated.

II. Ratio, Difference and Regression Estimation Methods

Cochran (1963), Deming (1961, 1963), Kisk (1965) and others have pointed out that ratio, difference and regression estimation methods are three traditional ways for utilizing auxiliary information to improve the sample estimates. The ratio method can further be analyzed into combined ratio, separate ratio and Hartley-Ross ratio methods. If auditors use stratification in selecting their samples, then there exist at least seven different methods to be used by auditors to obtain sample estimates. Following both Cochran (1963) and Hansen, Hurwitz and Madow (1953), the formula of these seven estimation methods used to estimate a population value can be defined as:

(A) Simple expansion: $Y_1 = \sum_h N_h \bar{Y}_h$

(B) The combined ratio estimate: $Y_2 = (\bar{Y}_{st}/\bar{X}_{st})\, X$

(C) The separate ratio estimate: $Y_3 = \sum_h (\bar{Y}_h/\bar{X}_h)\, X_h$

(D) The separate Hartley-Ross ratio estimate: $Y_4 = \sum_h \bar{r}'_h X_h$ (1)

(E) The combined regression estimate: $Y_5 = N[\bar{Y}_{st} + b(\bar{X} - \bar{X}_{st})]$

(F) The separate regression estimate: $Y_6 = \sum_h N_h [\bar{Y}_h + b_h(\bar{X}_h - \bar{X}_h)]$

(G) The combined difference estimate: $Y_7 = X + N(\overline{Y}_{st} - \overline{X}_{st})$

where $\overline{Y}_h$ = the sample average audited item value in the hth stratum

$\overline{X}_h$ = the sample average reported item value in the hth stratum

$\overline{Y}_{st}$ = sample over-all average audited item value

$\overline{X}_{st}$ = sample over-all average reported item value

X = total reported balance

N_h = total population item in the hth stratum

X_h = total reported balance in the hth stratum

$$\overline{r}'_h = \overline{r}_h + \frac{N_h(N_h - 1)}{(N_h - 1)N_h\overline{X}_h}(\overline{Y}_h - \overline{r}_h\overline{X}_h)$$

$$\overline{r} = \sum_{i=1}^{n} \frac{Y_{hi}}{X_{hi}}/n$$

N_h = total sample items in the hth stratum

$\overline{X} = X/N$

b = over-all regression coefficient

b_h = the regression slope associated with hth stratum.

The simple expansion method of equation (1A) estimates the total audited value without using the auxiliary information associated with client reported item values or its total balance. To improve the efficiency of estimated total audited value, six methods indicated in equation (1B) ~ (1G) have used the auxiliary information associated with clients' report value. Now the relationships among simple expansion me od, ratio method, regression method and difference method are discussed In general, the regression estimate includes simple expansion, the ratio and the difference methods as special cases. If the regression slope is taken as zero, then both Y_5 and Y_6 reduce to Y_1; if $b = \overline{Y}_{st}/\overline{X}_{st}$ and $b_h = \frac{\overline{Y}_h}{\overline{X}_h}$ then Y_5 and Y_6 reduce to Y_2 and Y_3, respectively; if $b = 1$, then Y_5

reduces to Y_7. Cochran (1963, 203-204) has discussed the relative advantage between the separate estimate and the combined estimate. In general, the combined estimate is used when the sample size is small and the separate estimate is used when the population regression coefficients (or ratios) differ from stratum to stratum.

The ratio estimate is generally biased and therefore, the unbiased ratio estimates have been derived by Hartley-Ross (1954) and Quenouille (1956). The spearate Hartley-Ross estimate is defined in equation (1D), and the Quenouille's bias adjustment method will be discussed in the next section. It is worth while to note that there exist some tradeoff between variance and bias when an unbiased ratio estimate is used.

The mean square error is used to determine the relative advantage of different estimations are discussed. Cochran (1963) has shown that the mean square error [MSE] measure is an important indicator for evaluating alternative estimate methods in sampling survey. The MSE of a sample estimate can be defined as:

$$\begin{aligned} \text{MSE } (\hat{\mu}) &= E(\hat{\mu} - \mu)^2 \\ &= E(\hat{\mu} - m)^2 + (m - \mu)^2 \qquad (2) \\ &= (\text{Variance of } \hat{\mu}) + (\text{bias})^2 \end{aligned}$$

where μ is true population mean; $\hat{\mu}$ is sample estimate of the population mean; m is the expected value of $\hat{\mu}$. The bias, $m - \mu$, is generally caused by the estimate associated with ratio or by the measurement error.

The variance of Y_1, Y_2, Y_3, Y_4, Y_5, Y_6 and Y_7 as indicated in equations (1A~1G) is defined as follows:

(i) the variance of simple expansion estimate can be defined as

$$\mathrm{Var}(Y_1) = \sum_h \frac{N_h^2(1 - f_h)}{N_h} S_{yh} \qquad (3)$$

where $f_h = n_h/N_h$

(ii) the variance associated with ratio estimates can be defined as

$$Var(Y_2) = \sum_h \frac{N_h^2(1 - f_h)}{N_h}(S_{yh}^2 + R^2 S_{\chi h}^2 - 2R\rho_h S_{yh} S_{\chi h}) \quad (4)$$

$$Var(Y_3) = \sum_h \frac{N_h^2(1 - f_h)}{N_h}(S_{yh}^2 + R_h^2 S_{\chi h}^2 - 2R_h \rho_h S_{yh} S_{\chi h}) \quad (5)$$

$$Var(Y_4) = \sum_h \frac{N_h^2}{N_h}(S_{yh}^2 + \bar{r}_{ph} S_{\chi h}^2 - 2\bar{r}_{ph} S_{yh.\chi h}) + \sum_h \frac{N_h^2}{N_h(N_h - 1)}[S_{rh}^2 S_{\chi h}^2 + S_{rh.\chi h}^2] \quad (6)$$

where $\bar{r}_{ph}$ and S_{rh}^2 are the population mean and variance of r_{hi} and $S_{rh\,.\chi h}^2$ is population covariance of r_{hi} and χ_{hi}.

(iii) the variance of two alternative regression methods can be defined as

$$Var(Y_5) = \frac{\sum_h N_h^2 W_h^2(1 - f_h)}{N_h}(S_{yh}^2 - 2bS_{y\chi h} + b^2 S_{\chi h}^2) \quad (7)$$

$$Var(Y_6) = \sum \frac{N_h^2 W_h^2(1 - f_h)}{N_h}(S_{yh}^2 - 2b_h S_{y\chi h} + b_h^2 S_{\chi h}^2) \quad (8)$$

where $W_h = N_h/N$, both Var (Y_5) and Var (Y_6) are minimized if the sample slope estimate is equal to true slope.

(iv) the variance of difference method can be defined as[1]

$$Var(Y_7) = \sum_h \frac{N_h^2 W_h^2(1 - f_h)}{N_h}(S_{yh}^2 - 2S_{y\chi h} + S_{\chi h}^2) \quad (9)$$

Compare equation (9) with equation (8), it is found that the difference method is a special case of combined regression method. In other words, the difference method has arbitrarily assumed the regression slope b to be one. Hence the biases associated with both ratio and difference

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[1]Both McCray (1973) and Ernst and Ernst (1975) have used the short-cut formula to estimate Var (Y_7).

methods are generally higher than those of the regression method if the sample size is large. In the following section the jackknife method is discussed.

III. Jackknife Method and Bias Reduction

Although Hartley-Ross's unbiased ratio method can reduce the bias, but it will increase the variance as indicated in equation (6). Following either Quenouille (1956), Durbin (1959), Cochran (1963), and Jones (1968), the jackknife method for estimating the population ratio can be defined as[2]

$$\hat{R}^q = K\hat{R} - (K - 1)E(\hat{R}_j) \tag{10}$$

where $\hat{R}^q$ = the jackknife ratio estimate

$\hat{R} = \Sigma y/\Sigma \chi$

$\hat{R}_j$ = the ordinary ratio $\Sigma y/\Sigma \chi$, computed from the sample after omitting the jth observation (or jth group). This is a combined jackknife method. It is clear that the separate jackknife method can be derived by integrating equation (1C) with equation (10). Both combined and separate jackknife methods will be explored further in the following section. Since the variance of $\hat{R}^q$ differs from that of $\hat{R}$ by terms of order of $1/n^2$, any increase in variance due to this adjustment for bias should therefore be neglible in moderately large samples. After the ratio $\hat{R}^q$ is obtained, the formula of either Y_2 or Y_3 can be used to estimate a population value.

From the analyses of this section and previous sections, it can be concluded that both the regression method and the jackknife method are potential methods to be used to reduce the bias of the estimates associated

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[2] This name was first used by Jones (1963). Miller (1974) has carefully reviewed the existing literature related to the jackknife method.

with sampling auditing. In the following section, data of 250 account receivable observations are used to do some simulation empirical studies.

IV. Sampling Procedure and Empirical Results

In most situations, an auditor will prefer not to obtain a simple random sample. Normally, this occurs when he believes some segments of the population differ significantly from the others with respect to certain characteristics. As an example, in the auditing of accounts receivable, the auditor will expect larger and more frequent errors in accounts with large balances than in those with small balances. In order to take this factor into consideration, the accounts receivable population needs to be stratified into a few subpopulations and selecting a separate random sample from each stratum. Related to stratification is to determine the number of strata and the size of a sample needed. Various criteria and methods can be used to determine them. Since the main purpose of our paper is not centering on the issue of stratification, we simply divide each of our two populations into two strata. That is, the first set of population consists of 250 accounts receivable reported year-end balances. The first 150 small accounts balances of the 250 accounts were classified as the first strata, the remaining 100 accounts were classified as the second strata. These 250 accounts balances were the division's data from a large manufacturing firm. Since the division's accounts receivable were not audited 100 percent last year, we were not able to obtain a complete set of populations of truly audited accounts receivable balances. However, based on some discussions at a meeting with the division's auditor and the division's accounting supervisor, we assumed that if all accounts were truly 100 percent audited, the first 150 accounts would contain slightly relative fewer and smaller

errors, and the last 100 accounts would contain relative more and larger errors.[3] Furthermore, based on the following observations, we assume that audited values were generally smaller than those originally reported on the accounts.

1. Some payments have already been made but the firm was slow in posting the accounts at the time of auditing.

2. Some goods have been returned by customers, but the division was also very slow in giving credit to the customers.

3. Some customers will automatically deduct the cash discount from remittance, even though the payments were made after the discount period. However, the firm was not in a position to strictly enforce (nor is willing to enforce) the discount period because of competitor's liberal discount period. This usually occurs when the customer has large volume of transactions with the division.

Since our populations is small and the frequently referred minimum sample size is 50, we will use 50 as our sample size. A simple random sample was taken from each of our two subpopulations of reported values. In the mean time, a corresponding sample was taken from each of the two subpopulations of audited values. Thus, one 50 pairs of reported and audited values were sampled from the first 150 accounts populations and another 50 pairs of reported and audited values were sampled from the next (and last) 100 accounts populations. The data obtained as described above were applied to the nine estimation methods. The sample data and the results were presented as follows:

The computation procedures were generally self-explained in the above equations. However, there are a few points need to be noted;

1. Following the general practice, the combined ratio obtained from the sample was used in computing the combined-ratio-total estimate variance.

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[3]However, in order to examine the effect of various characteristics of population on the sampling results, we also simulate a set of data with the assumption that the error rates in both strata are moderate and about equal. The results will be discussed later.

2. The jackknife combined ratio is computed as follows:

$$R_g = 100R_c - (100 - 1) \times \frac{\sum_{i=1}^{100} \hat{R}_{c(j)}}{100} \qquad (11)$$

where $\hat{R}_{c(j)}$ is the combined ratio computed from the sample after omitting the jth observation.

3. Jackknife separate ratio method is computed as follows:

$$R_g^{(i)} = 50R_s^{(i)} - (50 - 1) \left(\frac{\sum_{j=1}^{50} \hat{R}_{(j)}^{(i)}}{50}\right) \qquad (i = 1, 2) \qquad (12)$$

where $R_s^{(i)}$ is the separate ratio computed from the ith group; $\hat{R}_{(j)}^{(i)}$ is the separate ratio computed from the sample associated with the ith group after omitting the jth observation.

If $n_1 = 30$ and $n_2 = 50$ then the formula of equations (11) and (12) can easily be modified to calculate the related jackknife ratios.

4. The true variance for the separate Hartley-Ross ratio total estimate should be based on the $\bar{R}_{ph}$ and S_R (the mean of the subpopulation ratio and the standard variance of the subpopulation ratio).

$$\bar{R}_{P1} = \frac{\sum_{i=1}^{150} R_i}{150}; \qquad \bar{R}_{P2} = \frac{\sum_{i=151}^{250} R_i}{100}$$

However, the true subpopulation's ratio may not be available. Therefore, the variances based on the sample's data were also presented.[4]

$$\bar{r}_1 = \frac{\sum_{i=1}^{50} r_i}{50}; \qquad \bar{r}_2 = \frac{\sum_{i=1}^{50} r_i}{50}$$

To investigate the relative advantages among several alternative estimation methods in sampling auditing, a set of account receivables

[4] A relative_complicated formula associated with the unbiased sample estimate of Var(r'X) have been derived by Goodman and Hartley (1958, 497-450).

with 250 observations is used to do the empirical study. Assume that out of 250 observations being reported 81 have errors. The ratios between audited value and book value (AV/BV) which are different from zero, are listed in the Appendix A. The sample estimate of total value of accounts receivable (TARB) is $1,010,875.65 with a standard deviation of $87,035.42 using the simple expansion estimation method; the sample estimate of TARB is $1,117,075.80 with a standard deviation of $4,979.55 using the combined ratio method, and the sample estimate of TARB is $1,116,847.57 and its standard deviation is $5,078.80 using the separate ratio method to estimate TARB. Compare each sample estimate of TARB with the audited value, $1,124,258.00. It is found that each sample estimate of TARB is biased. To reduce the bias, both separate Hartley-Ross ratio and the jackknife ratio method are used to estimate the TARB. The Hartley-Ross ratio estimate of TARB is $1,116,368.45 and its standard deviation is $6,964.33; the combined jackknife ratio method sample estimate of TARB is $1,116,308.13 and its standard deviation is $4,999.69; the separate jackknife ratio method sample estimate of TARB is $1,117,347.36 and its standard deviation is $5,061.79. It is found that both the separate Hartley-Ross ratio method and jackknife ratio method do not reduce the bias.

Finally, both difference and regression estimation methods are used to estimate the TARB. As the difference method is used to estimate the TARB, the sample estimate is $1,120,554.25 and its standard deviation is $1,948.16; as the combined regression method is used to estimate the TARB, the sample estimate is $1,119,176.96 and its standard deviation is $1,948.16; as the separate regression method is used to estimate the TARB, the sample estimate is $1,119.172.48 and its standard deviation is $1,985.43. Since the regression slope is approximately equal to one (.9874), the result

obtained from the difference method is similar to the result obtained from the regression method. As both the ratio and regression slope obtained from the first stratum is similar to that obtained from the second stratum, the results associated with the combined methods are similar to those obtained from the separate methods. With regard to the results obtained from ratio methods are compared with those obtained from either regression or difference methods, the intercepts related to over-all regression and individual stratum regressions are significantly different from zero.[5] Hence, ratio methods are not appropriate for estimating the TARB. It would appear from the results listed in Table I, that those obtained from the difference method and the regression method are superior to those obtained from ratio methods [see Table I].

Hansen, Hurwitz and Madow (1953) [HHM] and Cochran (1963) have investigated the impact of bias on the statistical inference associated with sampling survey. They proposed that the ratio between bias and standard deviation (B/σ) be used to measure the importance of bias in sampling survey inferences. Estimated B and σ as indicated in Table I are used to estimate B/σ for all eight sample estimates and the results are listed in the last column of Table I. It is found that all B/σ estimates (except the Hartley-Ross ratio estimate) are larger than 1.30.[6] Therefore, the effect of biases on the inference associated with this set of data is not negligible. Both HHM and Cochran have pointed out that biases can be related either to the estimating ratio or to measurement error. The biases associated

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[5] For example, the intercept associated with combined regression method is $1,781.03.

[6] Since the Hartley-Ross ratio estimate is an inefficient estimator its standard deviation is $6,964.33.

with the estimating ratio will decrease as the sample size increases. However, the biases associated with measurement error will generally not be affected by the change of sample size.

Now, the relative ranking for 200 runs of simulation results associated with standard deviation (σ), bias (B) and B/σ are reported in Tables II-IV. In these tables, the relative rankings are reported in a descending order. From Table II, it is found that all 200 sample TARB estimates associated with simple expansion method have the highest standard deviation and all 200 sample TARB estimates associated with Hartley-Ross ratio method have second largest standard deviations. It is also found that sample TARB estimates associated with regression method have the lowest standard deviation. Table III indicates the relative ranking for bias among nine alternative estimation methods, it is found that sample TARB estimates obtained from regression methods have relative low bias. It is interesting to note that the sample TARB estimates associated with difference method have slightly smaller bias than those obtained from the regression methods. These results can be explained by the fact the regression estimates are not unbiased estimates unless the sample size is infinite [see Cochran (1963, 196-199)]. Table IV shows the relative ranking for B/σ among nine alternative estimation methods. The distribution of these relative rankings is similar to that indicated in the fifth column of Table I. To examine the effect of various characteristics of population on the sampling results, different error rates, i.e., high and low error rates are used to do simulations. It is found that the results of relative ranking σ, B and B/σ associated with both high and low error rates cases are similar to those presented in Tables II-IV.

From the simulation results associated with σ and B, it can be concluded that both regression and difference methods are suitable for audit sampling estimation. It is also found that both jackknife ratio and Hartley-Ross ratio methods have not substantially reduced the bias associated with sample TARB estimates. If the B/σ criteria is used, then both simple expansion and Hartley-Ross ratio methods become two relative desirable methods.

V. Summary and Concluding Remarks

Nine alternative estimation methods used in sampling survey have been reviewed. The conditions of using different estimation methods are also explored. To investigate the relative advantage of using alternative estimation methods in audit sampling, a set of accounts receivable data with 250 observations was used to do some empirical studies. It is found that the regression and difference methods were the most acceptable methods to be used in the sampling auditing. The problem of reducing bias and the importance of the index B/σ in audit sampling was also discussed in some detail.

The reason of both Hartley-Ross method and jackknife method not reducing the biases associated with accounts receivable sample estimates will be investigated in the future research. The effect of bias on the audit sampling inferences will also be explored by using simulation technique in the near future.

TABLE I

Summary Table

	Method	TARB	Standard Deviation (σ)	Bias (B)	B/σ
(1)	Simple Expansion Estimate	$1,010,875.65	$87,035.42	-113,382.35	-1.3027
(2)	Combined Ratio Estimate	1,117,075.84	4,979.55	-7,182.20	-1.4424
(3)	Combined Jackknife Ratio Estimate	1,116,308.13	4,999.69	-7,949.87	-1.5910
(4)	Separate Ratio Estimate	1,116,849.57	5,078.81	-7,408.43	-1.4587
(5)	Separate Jackknife Ratio Estimate	1,117,347.36	5,061.79	-6,910.64	-1.3653
(6)	Separate Hartley-Ross Estimate	1,116,368.45	6,964.33	-7,889.55	-1.133
(7)	Difference Estimate	1,120,554.25	1,948.16	-3,704.75	-1.9017
(8)	Combined Regression Estimate	1,119,176.96	1,895.44	-5,081.04	-2.6807
(9)	Separate Regression Estimate	1,119,177.47	1,895.42	-5,085.60	-2.6831

(Total audited value = $1,124,258.00)

TABLE II

Relative Ranking for σ

Ranking / Methods	1	2	3	4	5	6	7	8	9
1. Simple Expansion	200	0	0	0	0	0	0	0	0
2. Combined Ratio	0	0	129	6	8	57	0	0	0
3. Combined Jackknife Ratio	0	0	3	128	58	11	0	0	0
4. Separate Ratio	0	0	68	3	128	1	0	0	0
5. Separate Jackknife Ratio	0	0	0	63	6	131	0	0	0
6. Hartley-Ross Ratio	0	200	0	0	0	0	0	0	0
7. Combined Regression	0	0	0	0	0	0	2	198	0
8. Separate Regression	0	0	0	0	0	0	0	0	200
9. Difference	0	0	0	0	0	0	198	2	0

TABLE III

Relative Ranking for B

Ranking / Methods	1	2	3	4	5	6	7	8	9
1. Simple Expansion	194	1	0	0	0	0	0	0	5
2. Combined Ratio	0	2	23	35	39	35	19	39	8
3. Combined Jackknife Ratio	3	42	37	20	10	8	13	22	45
4. Separate Ratio	0	0	18	42	54	67	66	40	6
5. Separate Jackknife Ratio	0	22	10	19	68	40	17	11	13
6. Hartley-Ross Ratio	0	1	21	52	34	53	89	38	5
7. Combined Regression	0	26	49	25	13	16	16	29	25
8. Separate Regression	1	31	36	29	9	16	19	32	27
9. Difference	2	75	19	10	1	2	7	13	71

TABLE IV

Relative Ranking for B/σ

Ranking / Methods	1	2	3	4	5	6	7	8	9
1. Simple Expansion	23	18	25	22	10	4	4	16	78
2. Combined Ratio	0	0	2	19	56	51	52	19	1
3. Combined Jackknife Ratio	5	2	8	69	28	7	42	23	16
4. Separate Ratio	0	0	0	19	27	80	53	21	0
5. Separate Jackknife Ratio	0	0	1	33	69	48	34	13	2
6. Hartley-Ross Ratio	0	0	0	0	0	2	10	101	87
7. Combined Regression	42	88	48	13	2	1	1	4	1
8. Separate Regression	53	80	47	7	4	3	3	0	3
9. Difference	77	12	69	18	4	4	1	3	12

APPENDIX A

AV/BV Ratio

Observation Code	Ratio	Observation Code	Ratio
6	.9801	143	.9400
13	.9306	148	.9173
14	.8952	150	.9400
18	.8186	152	.9317
21	.9276	154	1.0150
22	.9687	156	.9449
23	.9421	158	.9040
25	.9710	161	.9115
32	.9760	162	.9209
35	.9310	164	1.0181
37	.9327	167	.9622
39	.9212	169	.8720
42	.9309	170	.9211
43	.9428	172	.9277
45	.9566	175	.9167
52	.9531	180	.8988
53	.9440	184	.8947
55	.9417	188	.9463
60	.9298	191	.8881
61	.9771	193	.9440
64	.9601	195	.9140
68	.9534	198	.8346
71	.9103	201	.9847
75	.9278	205	.9083
79	.9450	207	.9320
82	.8876	208	.9113
86	.9185	210	.9280
87	.9382	214	.9515
92	.9479	125	.9107
93	.9487	219	.9248
96	.9414	223	.9466
102	.9692	225	.9000
105	.9637	226	.9100
110	.9609	228	.8238
111	.9326	233	.9786
112	.9392	240	.9307
121	.9493	241	.9048
122	.9473	243	.8395
125	.9613	246	.9697
127	.9257	248	.9349
135	.9499		

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